

TMS Pr 8.42

A SPACECRAFT OF MASS 10,000 kg IS PARKED IN A CIRCULAR ORBIT 200 km ABOVE EARTH'S SURFACE. WHAT IS THE MINIMUM ENERGY REQUIRED TO PLACE IT IN A SYNCHRONOUS ORBIT. (NEGLECT THE MASS OF THE FUEL BURNED).

FIND $r_{\text{GEOSYNCHRONOUS}}$

$$T_{\text{GS}} = 24 \text{ hr} = 8.64 \times 10^4 \text{ s}$$

So

$$v_{\text{GS}} = \frac{2\pi r_{\text{GS}}}{T_{\text{GS}}}$$

APPLYING NSL

$$\frac{GM_{\oplus} m}{r_{\text{GS}}^2} = m \frac{v_{\text{GS}}^2}{r_{\text{GS}}}$$

$$GM_{\oplus} = r_{\text{GS}} \left(\frac{4\pi^2 v_{\text{GS}}^2}{T_{\text{GS}}^2} \right)$$

GIVING

$$r_{\text{GS}} = \left(\frac{GM_{\oplus} T_{\text{GS}}^2}{4\pi^2} \right)^{1/3} = \left(\frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24}) (86400)^2}{4\pi^2} \right)^{1/3}$$

$$r_{\text{GS}} = (7.54 \times 10^{22})^{1/3}$$

$$\boxed{r_{\text{GS}} = 4.23 \times 10^7 \text{ m}} \quad \text{RADIUS OF A GEOSYNCHRONOUS ORBIT}$$

THE ENERGY OF ANY ORBIT $\ll E_{\text{ORBIT}} = -\frac{k}{2a}$ (TMS 8.42)

FOR LOW ORBIT:

$$E_{\text{LOW}} = -\frac{GM_{\oplus} m}{2r_{\text{LOW}}} = -\frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24}) (10,000)}{2(6370 + 200) \times 10^3}$$

$$E_{\text{LOW}} = -3.04 \times 10^{11} \text{ J}$$



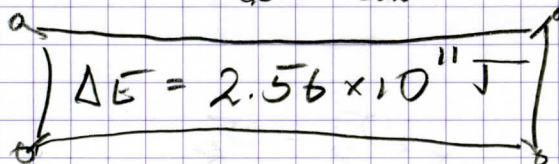
FOR THE GEOSYNCHRONOUS ORBIT

$$E_{GS} = -\frac{GM_{\oplus}m}{2r_{GS}} = -\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(10,000)}{2(4.23 \times 10^7)}$$

$$E_{GS} = -4.71 \times 10^{10} \text{ J}$$

THE ENERGY DIFFERENCE IS

$$\Delta E = E_{GS} - E_{LOW} = -4.71 \times 10^{10} - (-3.04 \times 10^{11})$$


$$\Delta E = 2.56 \times 10^{11} \text{ J}$$

ENERGY DIFFERENCE
BETWEEN ORBITS