TM5 Pr 8.42

A spacecraft of mass 10,000 kg is parked in a circular orbit 200 km above Earth's surface. What is the minimum energy required to place it in a synchronous orbit. (Neglect the mass of the fuel burned).

Find $r_{geosynchronous}$

$$\tau_{GS} = 24\,hr = 8.64 \times 10^4\,s$$

So

$$v_{GS} = \frac{2\pi r_{GS}}{\tau_{GS}}$$

Applying NSL

$$\frac{GM_\oplus m}{r_{GS}^2} = m\frac{v_{GS}^2}{r_{GS}}$$

$$GM_\oplus = r_{GS}\left(\frac{4\pi^2 r_{GS}^2}{\tau_{GS}^2}\right)$$

Giving

$$r_{GS} = \left(\frac{GM_\oplus \tau_{GS}^2}{4\pi^2}\right)^{1/3} = \left(\frac{(6.67\times10^{-11})(5.98\times10^{24})(86400)^2}{4\pi^2}\right)^{1/3}$$

$$r_{GS} = (7.54\times10^{22})^{1/3}$$

$$\boxed{r_{GS} = 4.23\times10^7\,m}$$ Radius of a geosynchronous orbit

The energy of any orbit is $E_{orbit} = -\frac{k}{2a}$ (TM5 8.42)

For low orbit:

$$E_{low} = -\frac{GM_\oplus m}{2r_{low}} = -\frac{(6.67\times10^{-11})(5.98\times10^{24})(10,000)}{2(6370+200)\times10^3}$$

$$E_{low} = -3.04\times10^{11}\,J$$

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FOR THE GEOSYNCHRONOUS ORBIT

$$E_{GS} = -\frac{GM_{\oplus}m}{2r_{GS}} = -\frac{(6.67\times10^{-11})(5.98\times10^{24})(10,000)}{2(4.23\times10^{7})}$$

$$E_{GS} = -4.71\times10^{10}\ J$$

THE ENERGY DIFFERENCE IS

$$\Delta E = E_{GS} - E_{LOW} = -4.71\times10^{10} - (-3.04\times10^{11})$$

$$\boxed{\Delta E = 2.56\times10^{11}\ J}$$

ENERGY DIFFERENCE BETWEEN ORBITS